



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Postscript to Note on a Point in Vulgar Fractions.

BY J. J. SYLVESTER.

LET ϕx represent $x^2 - x + 1$, $\phi^n c$ will then be the general term of the “limiting sorites” whose first term is c , for which, if we please, $1 - c$ may be substituted. The properties of the numbers $\phi^n c$ seem to be worthy of some attention. I confine my observations in what follows to the lowest of such series, viz. where $c = 2$ or -1 .

The first five terms in such series then become $\bar{1}$ or 2, 3, 7, 43, 1807, 3263443, of which all but 1807, which = 13.139, are prime numbers. Every term in the series must contain only factors of the form $6i + 1$, and this, joined to the fact that a prime factor which has once appeared in any term can never reappear in any other, favors a tendency, so to say, of the numbers to remain primes, or at all events, to be of very limited frangibility into a product of primes.

It is easy to determine whether any proposed prime can occur as a factor of any term whatever in the series; for taking that number, say p , as a modulus, if r is a remainder of any term to that modulus, the remainder of the next term will be $r^2 - r + 1$, and as soon as any remainder reappears the series of remainders becomes periodic; so that necessarily in less than the number p of remainders, if p does divide any term of the sorites, we must arrive at a remainder zero, subsequent to which all the remainders are unity. I give the remainders and periods in the annexed table for all values of p of the form $6i + 1$ up to 139, from which it will be seen that, under that limit, 13 and 73 are the only prime numbers which are contained as factors in the terms of the series.

p	Remainders of $\phi^n(2)$ to modulus p .
2	0.
3	2, 0.
7	2, 3, 0.
13	2, 3, 7, 4, 0.
19	2, 3, 7, 5; 2, 3, 7, 5;
31	2, 3, 7, 12, 9, 11, 18, 28, 13; 2, 3, 7,, 13;
37	2, 3, 7; 6, 31; 6, 31;
43	2, 3, 7, 0.
61	2, 3, 7, 43, 38, 4, 13, 35, 32; 17, 29, 20, 15, 28, 25, 52, 30;
67	2, 3; 7, 43, 65; 7, 43, 65;
73	2, 3, 7, 43, 55, 51, 69, 21, 56, 15, 65, 0.
79	2, 3, 7; 43, 69, 32, 45, 6, 31, 61, 27, 71, 73;
97	2, 3, 7, 43, 61; 72, 69, 37; 72, 69, 37;
103	2, 3; 7, 43, 56, 94, 91, 54, 82, 51, 79, 86, 101; 7, 43,;
109	{ 2, 3, 7, 43, 63, 92, 89, 94, 23, 71, 66, 40, 35, 101, 73, 25, 56, 29, 50, 53; 32, 12, 24, 8; 32, 12, 24, 8;
127	2, 3, 7, 43, 29, 51, 11; 111, 19, 89, 86, 72, 33, 41, 117;
139	2, 3, 7, 43, 0.
151	2, 3, 7, 43, 146; 31, 25, 148, 13, 6;
157	2, 3; 7, 43, 80, 41, 71, 104, 37, 77, 44, 9, 73, 76, 49, 155;
163	2, 3; 7, 43, 14, 20, 55, 37, 29, 161;
181	2, 3, 7, 43, 178, 13, 157, 58, 49, 0.
193	2; 3, 7, 43, 70, 6, 31, 159, 33, 92, 74, 192;
199	2, 3; 7, 43, 16, 42, 131, 116, 8, 57, 9, 73, 83, 41, 49, 164, 67, 45, 190, 91, 32, 197;